

Formula Sheet

1. **Complement rule:** $P(\overline{A}) = 1 - P(A)$.
2. **Additive law:** $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
3. **Multiplicative law:** $P(A \cap B) = P(A|B)P(B) = P(B|A)P(A)$.
4. **Conditional Probability:** $P(A|B) = P(A \cap B)/P(B)$.
5. **Independence:** Events A and B are independent, if and only if $P(A|B) = P(A)$ or $P(B|A) = P(B)$.
6. **Law of Total Probability:** $P(A) = P(A|B)P(B) + P(A|\overline{B})P(\overline{B})$.
7. **Bayes' rule:** $P(B|A) = \frac{P(A|B)P(B)}{P(A)} = \frac{P(A|B)P(B)}{P(A|B)P(B) + P(A|\overline{B})P(\overline{B})}$.
8. **Fundamental Theorem of Counting:** $N_1 \cdot N_2 \cdots N_k$.
9. **Permutation** $P_r^n \equiv \frac{n!}{(n-r)!} = n(n-1) \cdots (n-r+1)$.
10. **Combination** $C_r^n \equiv \binom{n}{r} = \frac{n!}{r!(n-r)!}$.

The following results are for discrete random variables:

11. **Mean:** $E(X) = \mu = \sum_{\text{all possible } x} xP(X=x)$. $E[g(X)] = \sum_{\text{all possible } x} g(x)P(X=x)$.
12. **Variance:** $\sigma^2 = \text{Var}(X) = E[(X - \mu)^2] = E(X^2) - [E(X)]^2$.
13. **Standard deviation:** $\sigma = \sqrt{\sigma^2}$
14. **Binomial:** $X \sim b(n, p)$, $p_X(x) = \binom{n}{x} p^x (1-p)^{n-x}$, $x = 0, 1, \dots, n$. $E(X) = np$, and $\text{Var}(X) = np(1-p)$.
15. **Geometric:** $X \sim \text{geo}(p)$, $p_X(x) = (1-p)^{x-1}p$, $x = 1, 2, \dots$. $E(X) = \frac{1}{p}$, and $\text{Var}(X) = \frac{1-p}{p^2}$.
16. **Hypergeometric:** $X \sim \text{hyper}(N, n, r)$. $p_X(x) = \frac{\binom{r}{x} \binom{N-r}{n-x}}{\binom{N}{n}}$, $y \leq r$ and $n-y \leq N-r$. $E(X) = n(\frac{r}{N})$, and $\text{Var} = n(\frac{r}{N})(\frac{N-r}{N})(\frac{N-n}{N-1})$.
17. **Poisson:** $X \sim \text{Poisson}(\lambda t)$. $p_X(x) = \frac{e^{-\lambda t} (\lambda t)^x}{x!}$, $x = 0, 1, \dots$. $E(X) = \text{Var}(X) = \lambda t$.